

Optimal Design of fMRI Stimuli for Impulse Response Estimation

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Introduction: For a linear time-invariant (LTI) system, the output $y(t)$ corresponding to an arbitrary input $f(t)$ is determined by the impulse response function $h(t)$. Accurate determination of $h(t)$ is obviously desirable. However, the accuracy of the estimate for $h(t)$ is dependent on the nature of $f(t)$. We consider how to select the input stimulus function $f(t)$ in order to obtain the most accurate estimate for the impulse response function (IRF).

Deconvolution of fMRI Time Series Data: For an LTI system, the output $y(t)$ is related to the input $f(t)$ through the convolution integral (1):

$$y(t) = \int_0^t f(\tau)h(t-\tau)d\tau.$$

Assuming that the IRF is essentially zero for time lags greater than p , this is approximated by:

$$y_n = \sum_{m=0}^p f_{n-m}h_m, \quad n \geq p.$$

For fMRI data, the measurement will be modeled by a constant plus linear trend plus noise, in addition to the signal:

$$Z_n = \beta_0 + \beta_1 n + h_0 f_n + h_1 f_{n-1} + \dots + h_p f_{n-p} + \varepsilon_n,$$

or, in matrix notation: $Z = X\beta + \varepsilon$, where $\beta = [\beta_0 \ \beta_1 \ h_0 \ \dots \ h_p]^T$, and $\varepsilon_n \stackrel{iid}{\sim} N(0, \sigma^2)$. The linear regression problem is to estimate $b = \hat{\beta}$ by minimizing the error sum of squares between the data Z and the fit $\hat{Z} = Xb$:

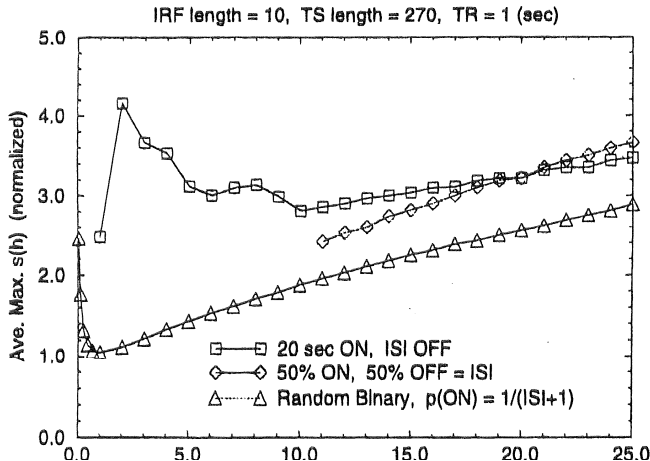
$$SSE = \sum_{i=1}^n (Z_i - \hat{Z}_i)^2 = (Z - \hat{Z})^t (Z - \hat{Z})$$

$$b = (X^t X)^{-1} X^t Z.$$

Error Analysis: For the above linear regression model, the variance-covariance matrix for the regression coefficients is given by (2):

$$s^2(b) = \frac{SSE}{N - 2p - 2} \cdot (X^t X)^{-1}$$

The std. dev. for the k th IRF coefficient is $s(h_k)$ = square root of the corresponding diagonal element of the $s^2(b)$



matrix. Assuming constant error variance σ^2 , the accuracy for the IRF estimate is determined by the structure of the experimental design matrix X , which depends on $f(t)$. This allows different candidate functions $f(t)$ to be evaluated numerically, *prior* to the fMRI experiment, for optimum estimation accuracy, as indicated in Figure 1.

Experimental Results: The subject performed the alternating finger tapping paradigm. Results for a particular voxel using a block design stimulus are presented in Figure 2, and for the same voxel using a random binary stimulus are presented in Figure 3. These results are typical for those voxels where activation was detected, and is consistent with the above numerical analysis.

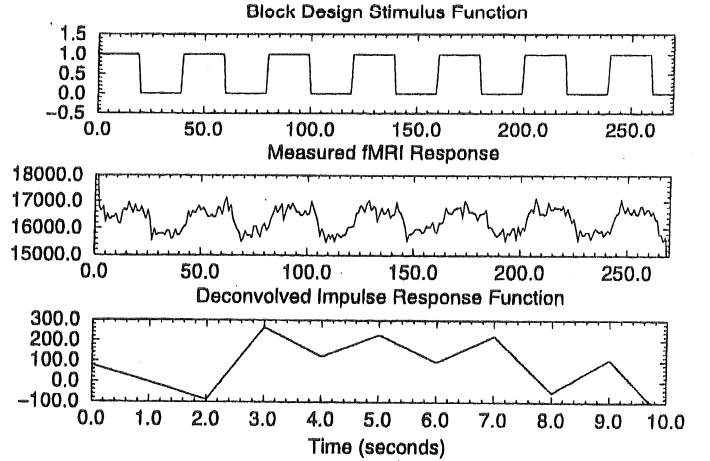


Figure 2. Block Design (20 sec. ON, 20 sec. OFF)

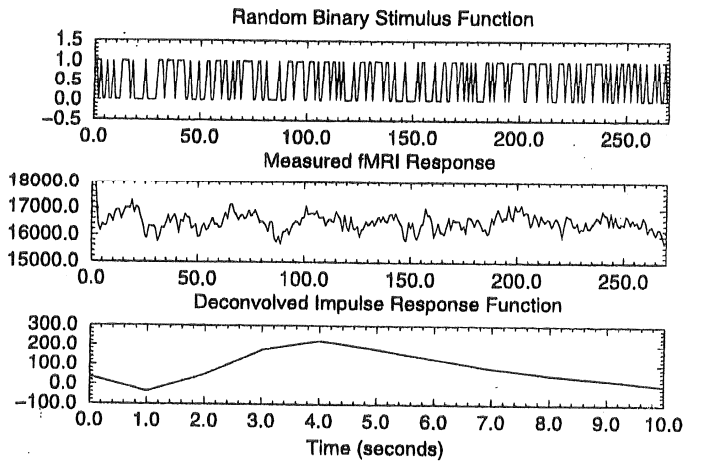


Figure 3. Random Binary ($p(\text{ON}) = p(\text{OFF}) = 0.5$)

Conclusion: Both numerical and experimental results indicate that proper design of the input stimulus function is critically important in obtaining an accurate estimate of the system impulse response function.

REFERENCES:

1. F. Stremmer, *Fourier Methods of Signal Analysis*, Lecture Notes. (1974)